



SUMMER VACATION HOLIDAY HOME-WORK

CLASS: XII

SUBJECT: APPLIED MATHEMATICS

1. Construct a 2×3 matrix $B = [b_{ij}]$ whose element b_{ij} is given by $|2i - 3j|$.

2. Write the number of all possible matrices of order 2×3 with each entry 5, 7 or 8.

3. Given $3 \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, find the values of x, y, z and w .

4. Find a matrix A such that $2A - 3B + 5C = 0$, where $B = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 0 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 3 & 2 \\ 0 & -5 & -1 \end{bmatrix}$.

5. If $A = [1, -2, 5]$, $B = [3, 0, -4]$ and $C = [-2, 7, 0]$, then find $2A + B - 2C$.

6. Find $x + y + z$, if $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.

7. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then find the value of $A^2 - 4A - 5I$.

8. Write the following as a single matrix: $\begin{bmatrix} 3 & 2 & 5 \\ 7 & -4 & 0 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 2 & -1 \\ 3 & 5 \end{bmatrix} - \begin{bmatrix} 7 & -8 \\ 5 & 9 \end{bmatrix}$.

9. If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $B = [-2 \quad -1 \quad -4]$, verify that $(AB)' = B'A'$.

10. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & x \\ -2 & 2 & -1 \end{bmatrix}$ is a matrix satisfying $AA' = 9I_3$, find x .

11. Using the properties of determinants, prove that

$$(i) \begin{vmatrix} 5a & -2a+b & -2c+a \\ -2b+a & 5b & -2b+c \\ -2c+a & -2c+b & 5c \end{vmatrix} = 12(a+b+c)(ab+bc+ca). \quad (ii) \begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$

12. Using properties of determinants, solve the following equations for x .

$$(i) \begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0. \quad (ii) \begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0.$$

13. Solve the following system of linear equations by Cramer's rule:

$$\begin{aligned} 6x + y - 3z - 5 &= 0 \\ \text{(i) } x + 3y - 2z &= 5 \\ 2x + 4z - 8 &= -7 \end{aligned}$$

$$\begin{aligned} \text{(ii) } \frac{2}{x} + \frac{3}{y} &= 2 \\ \frac{5}{x} + \frac{8}{y} &= \frac{31}{6} \end{aligned}$$

14. It is 7:00 pm currently. What time (in am or pm) will be in next 1505 hours?

15. Find the last digit of 17^{17} .

16. Three varieties A, B and C of rice are mixed together in the ratio 4:1:1 respectively. The cost price of rice B is ₹82 per kg and that of rice C is ₹90 per kg. If the price of the mixture is ₹94 per kg, then find the price per kg of rice A.

17. How many kg of sugar costing ₹45 per kg must be mixed with 30 kg sugar costing ₹35 per kg so that there may be a gain of 12% by selling the mixture at ₹47.04 per kg.

18. A boat takes 90 minutes less to travel 36 km downstream than to travel the same distance upstream. If the speed of the boat in still water is 10 km/hr, find the speed of the stream.

19. A boat covers 4 km against the stream in 1 hour and cover the same distance in the direction of stream in 40 minutes. How long will it take to go 10 km in still water?

20. Two pipes A and B can fill in 30 minutes and 45 minutes respectively. Both pipes A and B are opened together for some time and then pipe B is turned off. If the tank is filled in 20 minutes, then find after how many minutes the pipe B is turned off?

21. Three pipes A, B and C can fill a tank in 72 minutes. If all the three pipes remain opened for 36 minutes and then pipe C is closed, it took 1 hour more to fill the tank by pipes A and B. Find the time required to fill the tank by pipe C alone.

22. In a 200 m race, A can give a start of 31 m to B and a start of 28 m to C. In a race of 350 m, how much start can B give to C?

23. A can run 1 km in 4 minutes 54 seconds and B in 5 minutes. How many metres start can A give B in a km race so that the race may end in dead heat?

24. The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is atleast 61 cm, find the minimum length of the shortest side.

25. Solve the following inequality: $2y - 3 < y + 2 \leq 3y + 5$.
